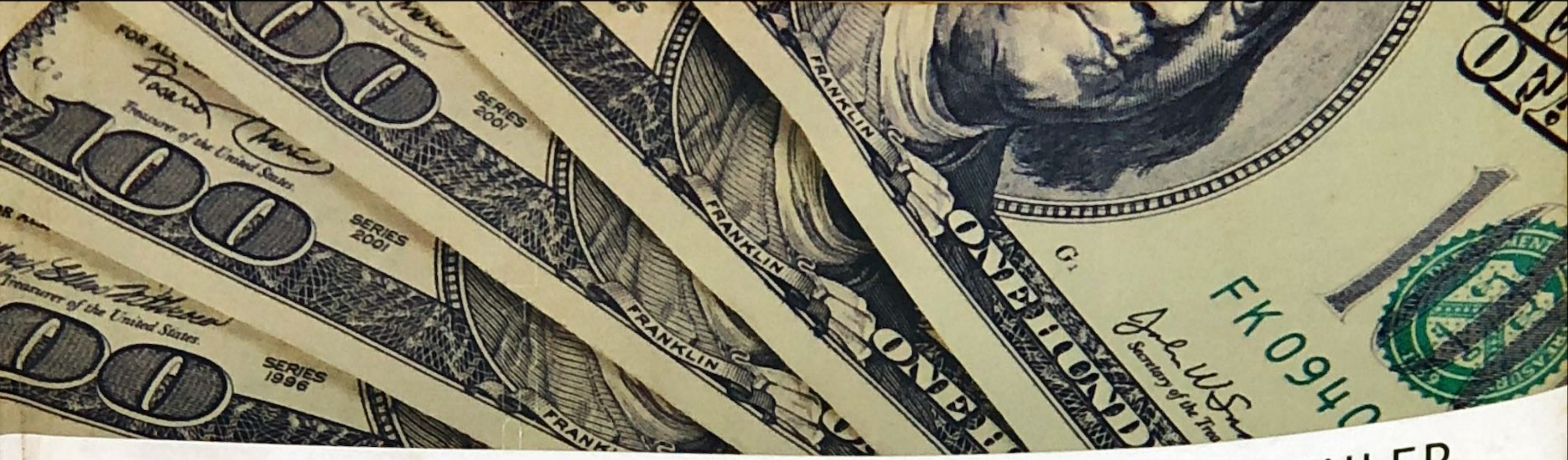

TIMOTHY J. BIEHLER

the mathematics of money

Math for Business and Personal Finance Decisions

Putting the Concepts in Context

Business Math requires certain core mathematical skills, such as working with percents and using algebra to solve basic equations. These skills can seem difficult to students, especially when presented in the abstract. The chapters of *The Mathematics of Money* are built around the concepts themselves, and the necessary math and algebra tools are introduced when the student needs them.

Core and Applications Chapters

This text is divided into *core* and *applications* chapters. The core chapters present key Business Math topics while at the same time developing basic algebra skills from scratch. The applications chapters then focus on applying and extending those skills to other business and finance topics. The core chapters lay the foundation for the course. The applications chapters are independent and can be taught in the order that works best for your course.

PART ONE: CORE MATHEMATICAL TOOLS

Chapter 1: Simple Interest
Chapter 2: Simple Discount
Chapter 3: Compound Interest
Chapter 4: Annuities
Chapter 5: Spreadsheets

PART TWO: SPECIFIC APPLICATIONS

Chapter 6: Investments
Chapter 7: Retirement Plans
Chapter 8: Mathematics of Pricing
Chapter 9: Taxes
Chapter 10: Consumer Mathematics
Chapter 11: International Business
Chapter 12: Financial Statements
Chapter 13: Insurance and Risk Management
Chapter 14: Evaluating Projected Cash Flows
Chapter 15: Payroll and Inventory
Chapter 16: Business Statistics

Using Algebra

Many Business Math texts avoid the use of algebra altogether. This text takes an algebraic approach, but it neither requires nor assumes any knowledge of algebra. Instead, *The Mathematics of Money* develops algebra skills from the ground up, alongside the presentation of the concepts. The text doesn't *require* algebra—it *teaches it* with the business and finance applications.

The Mathematics *of* Money

MATH *for* BUSINESS
and PERSONAL FINANCE DECISIONS

The Mathematics *of* Money

Math *for* Business
and Personal Finance Decisions

Timothy J. Biehler
Finger Lakes Community College

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Dedication

To Teresa, Julia, and Lily

About the Author

Timothy Biehler is an Assistant Professor at Finger Lakes Community College, where he has been teaching full time since 1999. He is a 2005 recipient of the State University of New York Chancellor's Award for Excellence in Teaching. Before joining the faculty at FLCC, he taught as an adjunct professor at Lemoyne College, SUNY–Morrisville, Columbia College, and Cayuga Community College.

Tim earned his B.A. in math and philosophy and M.A. in math at the State University of New York at Buffalo, where he was Phi Beta Kappa and a Woodburn Graduate Fellow. He worked for 7 years as an actuary in the life and health insurance industry before beginning to teach full time. He served as Director of Strategic Planning for Health Services Medical Corp. of Central New York, Syracuse, where he earlier served as Rating and Underwriting Manager. He also worked as an actuarial analyst for Columbian Financial Group, Binghamton, New York.

Tim lives in Fairport, New York, with his wife and two daughters.

Preface to Student

"Money is the root of all evil"—so the old adage goes. Whether we agree with that sentiment or not, we have to admit that if money is an evil, it is a *necessary* one. Love it or hate it, money plays a central role in the world and in our lives, both professional and personal. We all have to earn livings and pay bills, and to accomplish our goals, whatever they may be, reality requires us to manage the financing of those goals.

Sadly, though, financial matters are often poorly understood, and many otherwise promising ventures fail as a result of financial misunderstandings or misjudgments. A talented chef can open an outstanding restaurant, first rate in every way, only to see the doors closed as a result of financial shortcomings. An inventor with a terrific new product can nonetheless fail to bring it to market because of inadequate financing. An entrepreneur with an outstanding vision for a business can still fail to profit from it if savvier competition captures the same market with an inferior product but better management of the dollars and cents. And, on a more personal level, statistics continually show that "financial problems" are one of the most commonly cited causes of divorce in the United States.

Of course nothing in this book can guarantee you a top-rated restaurant, world-changing new product, successful business, or happy marriage. Yet, it is true that a reasonable understanding of money matters can certainly be a big help in achieving whatever it is you want to achieve in this life. It is also true that mathematics is a tool essential to this understanding. The goal of this book is to equip you with a solid understanding of the basic mathematical skills necessary to navigate the world of money.

Now, unfortunately (from my point of view at least), while not everyone would agree that money is root of all evil, it is not hard to find people who believe that mathematics is. Of course while some students come to a business math course with positive feelings toward the subject, certainly many more start off with less than warm and cozy feelings. Whichever camp you fall into, it is important to approach this book and the course it is being used for with an open mind. Yes, this is mathematics, but it is mathematics being put to a specific use. You may not fall in love with it, but you may find that studying math in the context of business and finance makes skills that once seemed painfully abstract do fall together in a way that makes sense.

Those who do not master money are mastered by it. Even if the material may occasionally be frustrating, hang in there! There is a payoff for the effort, and whether it comes easily or not, it will come if you stick with it.

WALKTHROUGH

The Mathematics of Money: Math for Business and Personal Finance is designed to provide a sound introduction to the uses of mathematics in business and personal finance applications. It has dual objectives of teaching both mathematics and financial literacy. The text wraps each skill or technique it teaches in a real-world context that shows you the *reason* for the mathematics you're learning.

HOW TO USE THIS BOOK

This book includes several key pedagogical features that will help you learn the skills needed to succeed in your course. Watch for these features as you read, and use them for review and practice.

FORMULAS
Core formulas are presented in formal, numbered fashion for easy reference.

EXAMPLES
Examples, using realistic businesses and situations, walk you through the application of a formula or technique to a specific, realistic problem.

DEFINITIONS
Core concepts are called out and defined formally and numbered for easy reference.

Throughout the text, key terms or concepts are set in color boldface italics within the paragraph and defined contextually.

The same logic applies to discount. If a \$500 note is discounted by \$20, it stands to reason that a \$5,000 note should be discounted by \$200. If a 6-month discount note is discounted by \$80, it stands to reason that a 12-month note would be discounted by \$160. Thus, modeling from what we did for interest, we can arrive at:

FORMULA 2.1
The Simple Discount Formula

$$D = MdT$$

where
D represents the amount of simple DISCOUNT for a loan,
M represents the MATURITY VALUE
d represents the interest DISCOUNT RATE (expressed as a decimal)
and
T represents the TERM for the loan

The simple discount formula closely mirrors the simple interest formula. The differences lie in the letters used (D rather than I and d in place of R, so that we do not confuse discount with interest) and in the fact that the discount is based on maturity value rather than on principal. Despite these differences, the resemblance between simple interest and simple discount should be apparent, and it should not be surprising that the mathematical techniques we used with simple interest can be equally well employed with simple discount.

Example 8.3.1 ***Ampersand Computers bought 12 computers from the manufacturer. The list price for the computers is \$895.00, and the manufacturer offered a 25% trade discount. How much did Ampersand pay for the computers?***

As with markdown, we can either take 25% of the price and subtract, or instead just multiply the price by 75% (found by subtracting 25% from 100%). The latter approach is a bit simpler: (75%)(\$895.00) = \$671.25 per computer. The total price for all 12 computers would be (12)(\$671.25) = \$8,055.

Even though it is more mathematically convenient to multiply by 75%, there are sometimes reasons to work things out the longer way. When the manufacturer bills Ampersand for this purchase, it would not be unusual for it to show the amount of this discount as a separate item. (The bill is called an *invoice*, and the net cost for an item is therefore sometimes called the *invoice price*.) In addition, the manufacturer may add charges for shipping or other fees on top of the cost of the items purchased (after the discount is applied). The invoice might look something like this:

International Difference Engines Invoice No. 1207
Box 404
Marbleburg, North Carolina 20252
INVOICE

Sold To:
Ampersand Computers
4539 North Henley Street
Olean, NY 14760

Date: May 28, 2007
Order #: 90125
Shipped: May 17, 2007

Quantity	Product #	Description	MSRP	Total
12	87435-G	IDE-Model G Laptop	\$895.00	\$10,740.00
			Subtotal	\$10,740.00
			LESS: 25% discount	(\$2,685.00)
			Net	\$8,055.00
			PLUS: Freight	\$350.00
			Total due	\$8,405.00

The discount may sometimes be written in parentheses (as it is in the example above) because this is a commonly used way of indicating a negative or subtracted number in

Definition 1.1.1
Interest is what a borrower pays a lender for the temporary use of the lender's money.

Or, in other words:

Definition 1.1.2
Interest is the "rent" that a borrower pays a lender to use the lender's money.

Interest is paid in addition to the repayment of the amount borrowed. In some cases, the amount of interest is spelled out explicitly. If we need to determine the total amount to be repaid, we can simply add the interest on to the amount borrowed.

One question that may come up here is how we know whether that 8½% interest rate quoted is the rate per year or the rate for the entire term of the loan. After all, the problem says the interest rate is 8½% for 3 years, which could be read to imply that the 8½% covers the entire 3-year period (in which case we would not need to multiply by 3).

The answer is that *unless it is clearly stated otherwise, interest rates are always assumed to be rates per year.* When someone says that an interest rate is 8½%, it is understood that this is the rate *per year*. Occasionally, you may see the Latin phrase *per annum* used with interest rates, meaning *per year* to emphasize that the rate is per year. You should not be confused by this, and since we are assuming rates are per year anyway, this phrase can usually be ignored.

The Simple Interest Formula

EXERCISES THAT BUILD BOTH SKILLS AND CONFIDENCE

Each section of every chapter includes a set of exercises that gives you the opportunity to practice and master the skills presented in the section. These exercises are organized in three groupings, designed to build your skills and your confidence so that you can master the material.

BUILDING FOUNDATIONS

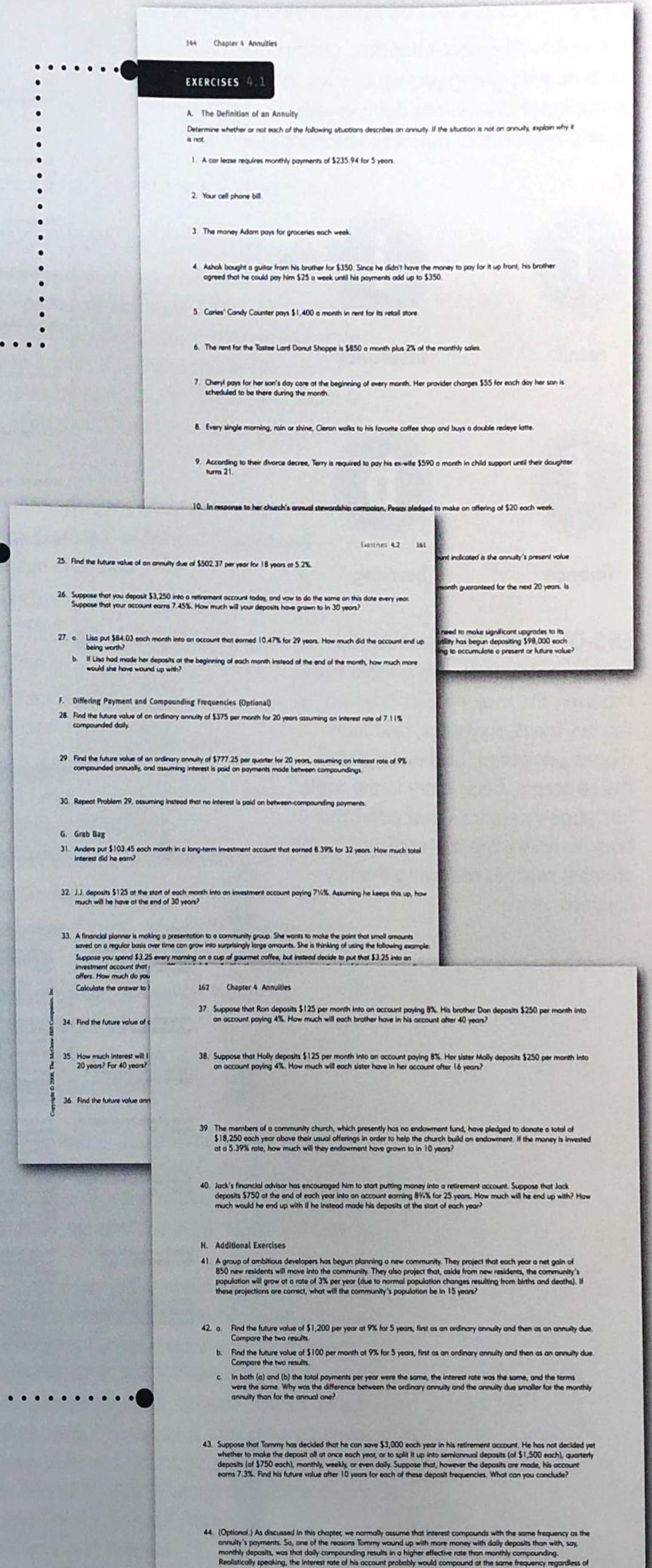
In each exercise set, there are several initial groupings of exercises under a header that identifies the type of problems that will follow and gives a good hint of what type of problem it is.

BUILDING CONFIDENCE

In each set there is also a grouping of exercises labeled "Grab Bag." These sections contain a mix of problems covering the various topics of the section, in an intentionally jumbled order. These exercises add an additional and very important layer of problem solving: identifying the type of problem and selecting an appropriate solution technique.

EXPANDING THE CONCEPTS

Each section's exercise set has one last grouping, labeled "Additional Exercises." These are problems that go beyond a standard problem for the section in question. This might mean that some additional concepts are introduced, certain technicalities are dealt with in greater depth, or that the problem calls for using a higher level of algebra than would otherwise be expected in the course.



ICONS

Throughout the **core chapters**, certain icons appear, giving you visual cues to examples or discussions dealing with several key kinds of business situations.

r retail

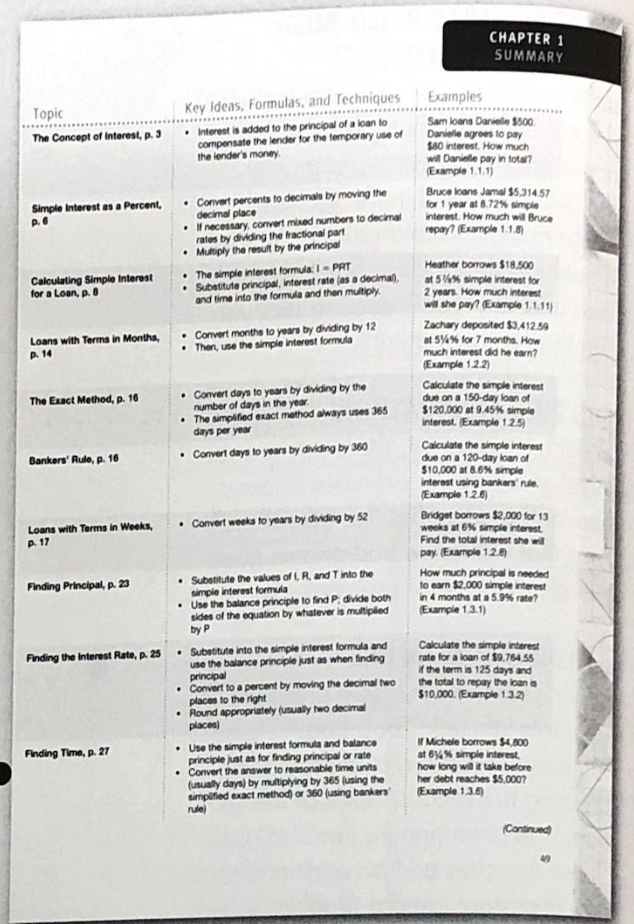
i insurance

f finance

b banking

END-OF-CHAPTER SUMMARIES

Each chapter ends with a table summarizing the major topics covered, the key ideas, formulas, and techniques presented, and examples of the concepts. Each entry in the table has page references that point you back to where the material was in the chapter, making reviewing the key concepts easier.



CHAPTER 1 SUMMARY

Topic	Key Ideas, Formulas, and Techniques	Examples
The Concept of Interest, p. 3	• Interest is added to the principal of a loan to compensate the lender for the temporary use of the lender's money.	Sam loans Danielle $500. Danielle agrees to pay $80 interest. How much will Danielle pay in total? (Example 1.1.1)
Simple Interest as a Percent, p. 6	• Convert percents to decimals by moving the decimal place • If necessary, convert mixed numbers to decimal rates by dividing the fractional part • Multiply the result by the principal	Bruce loans Jamal $5,314.57 for 1 year at 8.72% simple interest. How much will Bruce repay? (Example 1.1.8)
Calculating Simple Interest for a Loan, p. 8	• The simple interest formula: I = PRT • Substitute principal, interest rate (as a decimal), and time into the formula and then multiply.	Heather borrows $18,500 at 5⅞% simple interest for 2 years. How much interest will she pay? (Example 1.1.11)
Loans with Terms in Months, p. 14	• Convert months to years by dividing by 12 • Then, use the simple interest formula	Zachary deposited $3,412.59 at 5¼% for 7 months. How much interest did he earn? (Example 1.2.2)
The Exact Method, p. 16	• Convert days to years by dividing by the number of days in the year. • The simplified exact method always uses 365 days per year	Calculate the simple interest due on a 150-day loan of $120,000 at 9.45% simple interest. (Example 1.2.5)
Bankers' Rule, p. 16	• Convert days to years by dividing by 360	Calculate the simple interest due on a 120-day loan of $10,000 at 8.6% simple interest using bankers' rule. (Example 1.2.6)
Loans with Terms in Weeks, p. 17	• Convert weeks to years by dividing by 52	Bridget borrows $2,000 for 13 weeks at 6% simple interest. Find the total interest she will pay. (Example 1.2.8)
Finding Principal, p. 23	• Substitute the values of I, R, and T into the simple interest formula • Use the balance principle to find P; divide both sides of the equation by whatever is multiplied by P	How much principal is needed to earn $2,000 simple interest in 4 months at a 5.9% rate? (Example 1.3.1)
Finding the Interest Rate, p. 25	• Substitute into the simple interest formula and use the balance principle just as when finding principal • Convert to a percent by moving the decimal two places to the right • Round appropriately (usually two decimal places)	Calculate the simple interest rate for a loan of $9,764.55 if the term is 125 days and the total to repay the loan is $10,000. (Example 1.3.2)
Finding Time, p. 27	• Use the simple interest formula and balance principle just as for finding principal or rate • Convert the answer to reasonable time units (usually days) by multiplying by 365 (using the simplified exact method) or 360 (using bankers' rule)	If Michele borrows $4,800 at 6¼% simple interest, how long will it take before her debt reaches $5,000? (Example 1.3.6)

(Continued)

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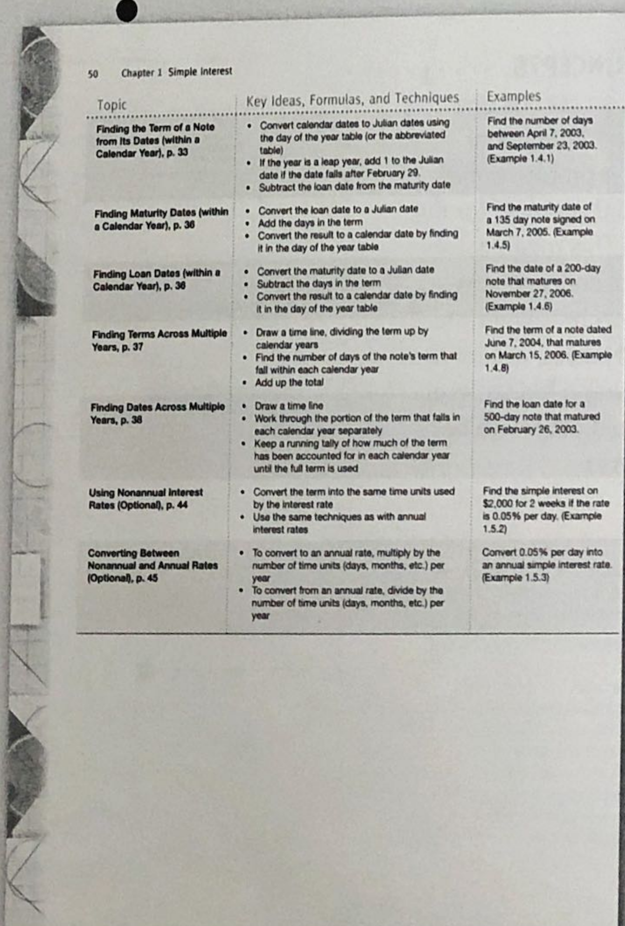

50 Chapter 1 Simple Interest

Topic	Key Ideas, Formulas, and Techniques	Examples
Finding the Term of a Note from Its Dates (within a Calendar Year), p. 33	• Convert calendar dates to Julian dates using the day of the year table (or the abbreviated table) • If the year is a leap year, add 1 to the Julian date if the date falls after February 29. • Subtract the loan date from the maturity date	Find the number of days between April 7, 2003, and September 23, 2003. (Example 1.4.1)
Finding Maturity Dates (within a Calendar Year), p. 36	• Convert the loan date to a Julian date • Add the days in the term • Convert the result to a calendar date by finding it in the day of the year table	Find the maturity date of a 135 day note signed on March 7, 2005. (Example 1.4.5)
Finding Loan Dates (within a Calendar Year), p. 36	• Convert the maturity date to a Julian date • Subtract the days in the term • Convert the result to a calendar date by finding it in the day of the year table	Find the date of a 200-day note that matures on November 27, 2006. (Example 1.4.6)
Finding Terms Across Multiple Years, p. 37	• Draw a time line, dividing the term up by calendar years • Find the number of days of the note's term that fall within each calendar year • Add up the total	Find the term of a note dated June 7, 2004, that matures on March 15, 2006. (Example 1.4.8)
Finding Dates Across Multiple Years, p. 38	• Draw a time line • Work through the portion of the term that falls in each calendar year separately • Keep a running tally of how much of the term has been accounted for in each calendar year until the full term is used	Find the loan date for a 500-day note that matured on February 26, 2003.
Using Nonannual Interest Rates (Optional), p. 44	• Convert the term into the same time units used by the interest rate • Use the same techniques as with annual interest rates	Find the simple interest on $2,000 for 2 weeks if the rate is 0.05% per day. (Example 1.5.2)
Converting Between Nonannual and Annual Rates (Optional), p. 45	• To convert to an annual rate, multiply by the number of time units (days, months, etc.) per year • To convert from an annual rate, divide by the number of time units (days, months, etc.) per year	Convert 0.05% per day into an annual simple interest rate. (Example 1.5.3)

Acknowledgments

Any project of this scope involves more people than the one whose name is printed on the cover, and this book is no exception.

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This book has undergone several rounds of reviews by instructors who are out there in the trenches, teaching this material. Each of them, with their thoughts and insights, helped improve this book.

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Kathy Boehler, *Central Community College*
Julliana R. Brey, *Cardinal Stritch University*
Bruce Broberg, *Central Community College*
Kelly Bruning, *Northwestern Michigan College*
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Tim Biehler

Brief Contents

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CORE MATHEMATICAL TOOLS 1