

TIMOTHY J. BIEHLER

the mathematics of money

Math for Business and Personal Finance Decisions

Putting the Concepts in Context

Business Math requires certain core mathematical skills, such as working with percents and using algebra to solve basic equations. These skills can seem difficult to students, especially when presented in the abstract. The chapters of *The Mathematics of Money* are built around the concepts themselves, and the necessary math and algebra tools are introduced when the student needs them.

Core and Applications Chapters

This text is divided into *core* and *applications* chapters. The core chapters present key Business Math topics while at the same time developing basic algebra skills from scratch. The applications chapters then focus on applying and extending those skills to other business and finance topics. The core chapters lay the foundation for the course. The applications chapters are independent and can be taught in the order that works best for your course.

PART ONE: CORE MATHEMATICAL TOOLS

- Chapter 1: Simple Interest
- Chapter 2: Simple Discount
- Chapter 3: Compound Interest
- Chapter 4: Annuities
- Chapter 5: Spreadsheets

PART TWO: SPECIFIC APPLICATIONS

- | | |
|------------------------------------|---|
| Chapter 6: Investments | Chapter 12: Financial Statements |
| Chapter 7: Retirement Plans | Chapter 13: Insurance and Risk Management |
| Chapter 8: Mathematics of Pricing | Chapter 14: Evaluating Projected Cash Flows |
| Chapter 9: Taxes | Chapter 15: Payroll and Inventory |
| Chapter 10: Consumer Mathematics | Chapter 16: Business Statistics |
| Chapter 11: International Business | |

Using Algebra

Many Business Math texts avoid the use of algebra altogether. This text takes an algebraic approach, but it neither requires nor assumes any knowledge of algebra. Instead, *The Mathematics of Money* develops algebra skills from the ground up, alongside the presentation of the concepts. The text doesn't *require* algebra—it *teaches it* with the business and finance applications.



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The Mathematics of Money

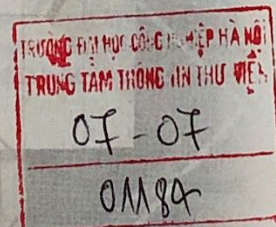
| MATH *for* BUSINESS
and PERSONAL FINANCE DECISIONS

The Mathematics of Money

Math for Business
and Personal Finance Decisions

Timothy J. Biehler

Finger Lakes Community College



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Dedication

To Teresa, Julia, and Lily

About the Author

Timothy Biehler is an Assistant Professor at Finger Lakes Community College, where he has been teaching full time since 1999. He is a 2005 recipient of the State University of New York Chancellor's Award for Excellence in Teaching. Before joining the faculty at FLCC, he taught as an adjunct professor at Lemoyne College, SUNY-Morrisville, Columbia College, and Cayuga Community College.

Tim earned his B.A. in math and philosophy and M.A. in math at the State University of New York at Buffalo, where he was Phi Beta Kappa and a Woodburn Graduate Fellow. He worked for 7 years as an actuary in the life and health insurance industry before beginning to teach full time. He served as Director of Strategic Planning for Health Services Medical Corp. of Central New York, Syracuse, where he earlier served as Rating and Underwriting Manager. He also worked as an actuarial analyst for Columbian Financial Group, Binghamton, New York.

Tim lives in Fairport, New York, with his wife and two daughters.

“Money is the root of all evil”—so the old adage goes. Whether we agree with that sentiment or not, we have to admit that if money is an evil, it is a *necessary* one. Love it or hate it, money plays a central role in the world and in our lives, both professional and personal. We all have to earn livings and pay bills, and to accomplish our goals, whatever they may be, reality requires us to manage the financing of those goals.

Sadly, though, financial matters are often poorly understood, and many otherwise promising ventures fail as a result of financial misunderstandings or misjudgments. A talented chef can open an outstanding restaurant, first rate in every way, only to see the doors closed as a result of financial shortcomings. An inventor with a terrific new product can nonetheless fail to bring it to market because of inadequate financing. An entrepreneur with an outstanding vision for a business can still fail to profit from it if savvier competition captures the same market with an inferior product but better management of the dollars and cents. And, on a more personal level, statistics continually show that “financial problems” are one of the most commonly cited causes of divorce in the United States.

Of course nothing in this book can guarantee you a top-rated restaurant, world-changing new product, successful business, or happy marriage. Yet, it is true that a reasonable understanding of money matters can certainly be a big help in achieving whatever it is you want to achieve in this life. It is also true that mathematics is a tool essential to this understanding. The goal of this book is to equip you with a solid understanding of the basic mathematical skills necessary to navigate the world of money.

Now, unfortunately (from my point of view at least), while not everyone would agree that money is root of all evil, it is not hard to find people who believe that mathematics is. Of course while some students come to a business math course with positive feelings toward the subject, certainly many more start off with less than warm and cozy feelings. Whichever camp you fall into, it is important to approach this book and the course it is being used for with an open mind. Yes, this is mathematics, but it is mathematics being put to a specific use. You may not fall in love with it, but you may find that studying math in the context of business and finance makes skills that once seemed painfully abstract do fall together in a way that makes sense.

Those who do not master money are mastered by it. Even if the material may occasionally be frustrating, hang in there! There is a payoff for the effort, and whether it comes easily or not, it will come if you stick with it.

The *Mathematics of Money: Math for Business and Personal Finance* is designed to provide a sound introduction to the uses of mathematics in business and personal finance applications. It has dual objectives of teaching both mathematics and financial literacy. The text wraps each skill or technique it teaches in a real-world context that shows you the reason for the mathematics you're learning.

HOW TO USE THIS BOOK

This book includes several key pedagogical features that will help you learn the skills needed to succeed in your course. Watch for these features as you read, and use them for review and practice.

FORMULAS

Core formulas are presented in formal, numbered fashion for easy reference.

EXAMPLES

Examples, using realistic businesses and situations, walk you through the application of a formula or technique to a specific, realistic problem.

DEFINITIONS

Core concepts are called out and defined formally and numbered for easy reference.

Throughout the text, key terms or concepts are set in color boldface italics within the paragraph and defined contextually.

The same logic applies to discount. If a \$500 note is discounted by \$20, it stands to reason that a \$5,000 note should be discounted by \$200. If a 6-month discount note is discounted by \$80, it stands to reason that a 12-month note would be discounted by \$160. Thus, modeling from what we did for interest, we can arrive at:

FORMULA 2.1
The Simple Discount Formula

$D = MdT$

where

- D represents the amount of simple DISCOUNT for a loan,
- M represents the MATURITY VALUE
- d represents the interest DISCOUNT RATE (expressed as a decimal)
- and
- T represents the TERM for the loan

The simple discount formula closely mirrors the simple interest formula. The differences lie in the letters used (D rather than I and d in place of R, so that we do not confuse discount with interest) and in the fact that the discount is based on maturity value rather than on principal. Despite these differences, the resemblance between simple interest and simple discount should be apparent, and it should not be surprising that the mathematical techniques we used with simple interest can be equally well employed with simple discount.

Example 8.3.1 Ampersand Computers bought 12 computers from the manufacturer. The list price for the computers is \$895.00, and the manufacturer offered a 25% trade discount. How much did Ampersand pay for the computers?

As with markdown, we can either take 25% of the price and subtract, or instead just multiply the price by 75% (found by subtracting 25% from 100%). The latter approach is a bit simpler: $(75\%)(\$895.00) = \671.25 per computer. The total price for all 12 computers would be $(12)(\$671.25) = \$8,055$.

Even though it is more mathematically convenient to multiply by 75%, there are sometimes reasons to work things out the longer way. When the manufacturer bills Ampersand for this purchase, it would not be unusual for it to show the amount of this discount as a separate item. (The bill is called an *invoice*, and the net cost for an item is therefore sometimes called the *invoice price*.) In addition, the manufacturer may add charges for shipping or other fees on top of the cost of the items purchased (after the discount is applied). The invoice might look something like this:

International Difference Engines

Box 404
Marbleburg, North Carolina 20252

Invoice No. 1207

Sold To:

Ampersand Computers
4539 North Henley Street
Olean, NY 14760

INVOICE

Date: May 28, 2007
Order #: 90125
Shipped: May 17, 2007

Quantity	Product #	Description	MSRP	Total
12	87435-G	IDE-Model G Laptop	\$895.00	\$10,740.00
Subtotal				\$10,740.00
LESS: 25% discount				(\$2,685.00)
Net				\$8,055.00
PLUS: Freight				\$350.00
Total due				\$8,405.00

The discount may sometimes be written in parentheses (as it is in the example above) because this is a commonly used way of indicating a negative or subtracted number in

Definition 1.1.1

Interest is what a borrower pays a lender for the temporary use of the lender's money.

Or, in other words:

Definition 1.1.2

Interest is the "rent" that a borrower pays a lender to use the lender's money.

Interest is paid in addition to the repayment of the amount borrowed. In some cases, the amount of interest is spelled out explicitly. If we need to determine the total amount to be repaid, we can simply add the interest on to the amount borrowed.

One question that may come up here is how we know whether that $8\frac{1}{2}\%$ interest rate quoted is the rate per year or the rate for the entire term of the loan. After all, the problem says the interest rate is $8\frac{1}{2}\%$ for 3 years, which could be read to imply that the $8\frac{1}{2}\%$ covers the entire 3-year period (in which case we would not need to multiply by 3).

The answer is that unless it is clearly stated otherwise, interest rates are always assumed to be rates per year. When someone says that an interest rate is $8\frac{1}{2}\%$, it is understood that this is the rate per year. Occasionally, you may see the Latin phrase *per annum* used with interest rates, meaning per year to emphasize that the rate is per year. You should not be confused by this, and since we are assuming rates are per year anyway, this phrase can usually be ignored.

The Simple Interest Formula

EXERCISES THAT BUILD BOTH SKILLS AND CONFIDENCE

Each section of every chapter includes a set of exercises that gives you the opportunity to practice and master the skills presented in the section.

These exercises are organized in three groupings, designed to build your skills and your confidence so that you can master the material.

BUILDING FOUNDATIONS

In each exercise set, there are several initial groupings of exercises under a header that identifies the type of problems that will follow and gives a good hint of what type of problem it is.

BUILDING CONFIDENCE

In each set there is also a grouping of exercises labeled "Grab Bag." These sections contain a mix of problems covering the various topics of the section, in an intentionally jumbled order. These exercises add an additional and very important layer of problem solving: identifying the type of problem and selecting an appropriate solution technique.

EXPANDING THE CONCEPTS

Each section's exercise set has one last grouping, labeled "Additional Exercises." These are problems that go beyond a standard problem for the section in question. This might mean that some additional concepts are introduced, certain technicalities are dealt with in greater depth, or that the problem calls for using a higher level of algebra than would otherwise be expected in the course.

EXERCISES 4.1

A. The Definition of an Annuity

Determine whether or not each of the following situations describes an annuity. If the situation is not an annuity, explain why it is not.

1. A car lease requires monthly payments of \$235.94 for 5 years.
2. Your cell phone bill.
3. The money Adam pays for groceries each week.
4. Ashok bought a guitar from his brother for \$350. Since he didn't have the money to pay for it up front, his brother agreed that he could pay him \$25 a week until his payments add up to \$350.
5. Carlen's Candy Counter pays \$1,400 a month in rent for its retail store.
6. The rent for the Tastee-Lord Donut Shoppe is \$850 a month plus 2% of the monthly sales.
7. Cheryl pays for her son's day care at the beginning of every month. Her provider charges \$55 for each day her son is scheduled to be there during the month.
8. Every single morning, rain or shine, Cleon walks to his favorite coffee shop and buys a double decaffe latte.
9. According to their divorce decree, Terry is required to pay his ex-wife \$590 a month in child support until their daughter turns 21.
10. In response to her church's annual stewardship campaign, Peggy pledged to make an offering of \$20 each week.

25. Find the future value of an annuity due of \$502.37 per year for 18 years at 5.2%.
26. Suppose that you deposit \$3,250 into a retirement account today, and now to do the same on this date every year. Suppose that your account earns 7.45%. How much will your deposits have grown to in 30 years?
27. a. Lisa put \$84.03 each month into an account that earned 10.47% for 29 years. How much did the account end up being worth?
b. If Lisa had made her deposits at the beginning of each month instead of the end of the month, how much more would she have wound up with?
- F. Differing Payment and Compounding Frequencies (Optional)
28. Find the future value of an ordinary annuity of \$375 per month for 20 years assuming an interest rate of 7.11% compounded daily.
29. Find the future value of an ordinary annuity of \$777.25 per quarter for 20 years, assuming an interest rate of 9% compounded annually, and assuming interest is paid on payments made between compounding.
30. Repeat Problem 29, assuming instead that no interest is paid on between-compounding payments.
- G. Grab Bag
31. Anders put \$103.45 each month in a long-term investment account that earned 8.39% for 32 years. How much total interest did he earn?
32. J.J. deposits \$125 at the start of each month into an investment account paying 7 1/8%. Assuming he keeps this up, how much will he have at the end of 30 years?
33. A financial planner is making a presentation to a community group. She wants to make the point that small amounts saved on a regular basis over time can grow into surprisingly large amounts. She is thinking of using the following example: Suppose you spend \$3.25 every morning on a cup of gourmet coffee, but instead decide to put that \$3.25 into an investment account that offers 7.45% interest. How much do you have after 30 years? Calculate the answer to 1 decimal place.
34. Find the future value of \$100 per month for 20 years at 6%.
35. How much interest will I have after 20 years? For 40 years?
36. Find the future value of \$100 per month for 20 years at 6%.

37. Suppose that Ron deposits \$125 per month into an account paying 8%. His brother Don deposits \$250 per month into an account paying 4%. How much will each brother have in his account after 40 years?
38. Suppose that Holly deposits \$125 per month into an account paying 8%. Her sister Molly deposits \$250 per month into an account paying 4%. How much will each sister have in her account after 16 years?
39. The members of a community church, which presently has no endowment fund, have pledged to donate a total of \$18,250 each year above their usual offerings in order to help the church build an endowment. If the money is invested at a 5.39% rate, how much will they endowment have grown to in 10 years?
40. Jack's financial advisor has encouraged him to start putting money into a retirement account. Suppose that Jack deposits \$750 at the end of each year into an account earning 8 1/8% for 25 years. How much will he end up with? How much would he end up with if he instead made his deposits at the start of each year?
- H. Additional Exercises
41. A group of ambitious developers has begun planning a new community. They project that each year a net gain of 850 new residents will move into the community. They also project that, aside from new residents, the community's population will grow at a rate of 3% per year (due to normal population changes resulting from births and deaths). If these projections are correct, what will the community's population be in 15 years?
42. a. Find the future value of \$1,200 per year at 9% for 5 years, first as an ordinary annuity and then as an annuity due. Compare the two results.
b. Find the future value of \$100 per month at 9% for 5 years, first as an ordinary annuity and then as an annuity due. Compare the two results.
c. In both (a) and (b) the total payments per year were the same, the interest rate was the same, and the terms were the same. Why was the difference between the ordinary annuity and the annuity due smaller for the monthly annuity than for the annual one?
43. Suppose that Tommy has decided that he can save \$3,000 each year in his retirement account. He has not decided yet whether to make the deposit at the end of each year, or to split it up into semiannual deposits (of \$1,500 each), quarterly deposits (of \$750 each), monthly, weekly, or even daily. Suppose that, however the deposits are made, his account earns 7.3%. Find his future value after 10 years for each of these deposit frequencies. What can you conclude?
44. (Optional) As discussed in this chapter, we normally assume that interest compounds with the same frequency as the annuity's payments. So, one of the reasons Tommy wound up with more money with daily deposits than with, say, monthly deposits, was that daily compounding results in a higher effective rate than monthly compounding. Realistically speaking, the interest rate of his account probably would compound at the same frequency regardless of how often Tommy makes his deposits. Rework Problem 43, this time assuming that, regardless of how often he makes his deposits, his account will pay 7.3% compounded daily.

ICONS

Throughout the **core chapters**, certain icons appear, giving you visual cues to examples or discussions dealing with several key kinds of business situations.



retail



insurance



finance



banking

END-OF-CHAPTER SUMMARIES

Each chapter ends with a table summarizing the major topics covered, the key ideas, formulas, and techniques presented, and examples of the concepts. Each entry in the table has page references that point you back to where the material was in the chapter, making reviewing the key concepts easier.

CHAPTER 1 SUMMARY

Topic	Key Ideas, Formulas, and Techniques	Examples
The Concept of Interest, p. 3	<ul style="list-style-type: none"> Interest is added to the principal of a loan to compensate the lender for the temporary use of the lender's money. 	Sam loans Danielle \$500. Danielle agrees to pay \$60 interest. How much will Danielle pay in total? (Example 1.1.1)
Simple Interest as a Percent, p. 6	<ul style="list-style-type: none"> Convert percents to decimals by moving the decimal place. If necessary, convert mixed numbers to decimal rates by dividing the fractional part. Multiply the result by the principal. 	Bruce loans Jamal \$5,314.57 for 1 year at 8.72% simple interest. How much will Bruce repay? (Example 1.1.8)
Calculating Simple Interest for a Loan, p. 8	<ul style="list-style-type: none"> The simple interest formula: $I = PRT$ Substitute principal, interest rate (as a decimal), and time into the formula and then multiply. 	Heather borrows \$18,500 at 5 1/4% simple interest for 2 years. How much interest will she pay? (Example 1.1.11)
Loans with Terms in Months, p. 14	<ul style="list-style-type: none"> Convert months to years by dividing by 12. Then, use the simple interest formula. 	Zachary deposited \$3,412.59 at 5 1/4% for 7 months. How much interest did he earn? (Example 1.2.2)
The Exact Method, p. 16	<ul style="list-style-type: none"> Convert days to years by dividing by the number of days in the year. The simplified exact method always uses 365 days per year. 	Calculate the simple interest due on a 150-day loan of \$120,000 at 9.45% simple interest. (Example 1.2.5)
Bankers' Rule, p. 16	<ul style="list-style-type: none"> Convert days to years by dividing by 360. 	Calculate the simple interest due on a 120-day loan of \$10,000 at 8.6% simple interest using bankers' rule. (Example 1.2.6)
Loans with Terms in Weeks, p. 17	<ul style="list-style-type: none"> Convert weeks to years by dividing by 52. 	Bridget borrows \$2,000 for 13 weeks at 6% simple interest. Find the total interest she will pay. (Example 1.2.8)
Finding Principal, p. 23	<ul style="list-style-type: none"> Substitute the values of I, R, and T into the simple interest formula. Use the balance principle to find P; divide both sides of the equation by whatever is multiplied by P. 	How much principal is needed to earn \$2,000 simple interest in 4 months at a 5.9% rate? (Example 1.3.1)
Finding the Interest Rate, p. 25	<ul style="list-style-type: none"> Substitute into the simple interest formula and use the balance principle just as when finding principal. Convert to a percent by moving the decimal two places to the right. Round appropriately (usually two decimal places). 	Calculate the simple interest rate for a loan of \$9,764.55 if the term is 125 days and the total to repay the loan is \$10,000. (Example 1.3.2)
Finding Time, p. 27	<ul style="list-style-type: none"> Use the simple interest formula and balance principle just as for finding principal or rate. Convert the answer to reasonable time units (usually days) by multiplying by 365 (using the simplified exact method) or 360 (using bankers' rule). 	If Michele borrows \$4,800 at 8 1/4% simple interest, how long will it take before her debt reaches \$5,000? (Example 1.3.6)

(Continued)

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50 Chapter 1 Simple Interest

Topic	Key Ideas, Formulas, and Techniques	Examples
Finding the Term of a Note from Its Dates (within a Calendar Year), p. 33	<ul style="list-style-type: none"> Convert calendar dates to Julian dates using the day of the year table (or the abbreviated table). If the year is a leap year, add 1 to the Julian date if the date falls after February 29. Subtract the loan date from the maturity date. 	Find the number of days between April 7, 2003, and September 23, 2003. (Example 1.4.1)
Finding Maturity Dates (within a Calendar Year), p. 36	<ul style="list-style-type: none"> Convert the loan date to a Julian date. Add the days in the term. Convert the result to a calendar date by finding it in the day of the year table. 	Find the maturity date of a 135-day note signed on March 7, 2005. (Example 1.4.5)
Finding Loan Dates (within a Calendar Year), p. 36	<ul style="list-style-type: none"> Convert the maturity date to a Julian date. Subtract the days in the term. Convert the result to a calendar date by finding it in the day of the year table. 	Find the date of a 200-day note that matures on November 27, 2006. (Example 1.4.6)
Finding Terms Across Multiple Years, p. 37	<ul style="list-style-type: none"> Draw a time line, dividing the term up by calendar years. Find the number of days of the note's term that fall within each calendar year. Add up the total. 	Find the term of a note dated June 7, 2004, that matures on March 15, 2006. (Example 1.4.8)
Finding Dates Across Multiple Years, p. 38	<ul style="list-style-type: none"> Draw a time line. Work through the portion of the term that falls in each calendar year separately. Keep a running tally of how much of the term has been accounted for in each calendar year until the full term is used. 	Find the loan date for a 500-day note that matured on February 26, 2003.
Using Nonannual Interest Rates (Optional), p. 44	<ul style="list-style-type: none"> Convert the term into the same time units used by the interest rate. Use the same techniques as with annual interest rates. 	Find the simple interest on \$2,000 for 2 weeks if the rate is 0.05% per day. (Example 1.5.2)
Converting Between Nonannual and Annual Rates (Optional), p. 45	<ul style="list-style-type: none"> To convert to an annual rate, multiply by the number of time units (days, months, etc.) per year. To convert from an annual rate, divide by the number of time units (days, months, etc.) per year. 	Convert 0.05% per day into an annual simple interest rate. (Example 1.5.3)

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Any project of this scope involves more people than the one whose name is printed on the cover, and this book is no exception.

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This book has undergone several rounds of reviews by instructors who are out there in the trenches, teaching this material. Each of them, with their thoughts and insights, helped improve this book.

Yvonne Alder, *Central Washington University–Ellensburg*

Kathy Boehler, *Central Community College*

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Tim Biehler

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PART one

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